Lesson 4-1: Congruent Figures

Congruent segments and angles

If I ask you what it means that two segments or two angles are congruent, you can tell me. But what if I asked you what it means that two triangles are congruent? What does that mean? Take a few moments and try to come up with a good definition for congruent triangles.

Congruent figures

For two figures to be considered congruent, they have to be exactly alike. If you could lay one on top of the other, they'd be a perfect match. Every side would be congruent and every angle would be congruent.

Naming Congruent Parts

Suppose we have two congruent triangles: $TJD \cong RCF$. The triangles **<u>must</u>** be named with corresponding vertices in order. Once we know which vertices correspond to which, we can then state which angles correspond and which sides correspond. Each vertex marks an angle. Each side is made of two vertices.

Thus, if we are told that two named polygons are congruent, we don't even need to see diagrams of them to be able to name all the congruent angles and sides.

To match vertices (and hence angles) I like to write one over the other. Then it is easy to see which vertices are paired. Using the example of $TJD \cong RCF$ we have:

T	J	D
R	C	F

Showing it this way helps us see which vertices are corresponding: $\angle T \cong \angle R, \angle J \cong \angle C \,\&\, \angle D \cong \angle F$

Wait a minute … it is easy to see how to match up the angles. How do we match up the sides? A side is created by two vertices right? Just take the vertices two at a time in order, starting from the left. When you hit the last one, just "join" it with the first. I like to draw lines under the sides to help me see it like this (using our prior example):

$TJD \cong RCF$ This makes it easier to see that $\overline{TJ} \cong \overline{RC}, \overline{JD} \cong \overline{CF} \,\&\, \overline{TD} \cong \overline{RF}$

These tricks help me identify corresponding parts much easier!

Example – Pg 182, Problem #2

If you try to go by the picture, it can be confusing. However if you simply go by the triangle congruence statement it is much easier: $\Delta EFG \cong \Delta HIJ$. One thing to be careful of here: you can't name angles by a single vertex because there are multiple angles at each vertex. Just go three at a time from the start. If you hit the end, "wrap-around" to the first and keep going:

Angles: $\angle EFG \cong \angle HIJ, \angle FGE \cong \angle IJH, \angle GEF \cong \angle JHI$

Lesson 4-1: Congruent Figures

Sides: $\overline{EF} \cong \overline{HI}, \overline{FG} \cong \overline{IJ}, \overline{GE} \cong \overline{JH}$

Example – Pg 182, Problems #4-12 even

4. $\overline{KJ} \cong \overline{CM}$
6. $\angle L \cong \angle B$
8. $\angle M \cong \angle J$
10. $\Delta KBJ \cong \Delta CLM$
12. $\Delta JKB \cong \Delta MCL$

Theorem 4-1

If two angles of one triangle are congruent to two angles of a different triangle, then the third angles are congruent.

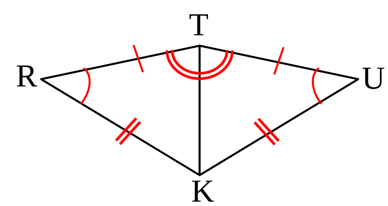
This theorem follows from the Triangle Angle-Sum Theorem. But be careful with this! If I have two triangles with congruent angles, does that mean the triangles are congruent? No! All the angles can be congruent but one triangle may be a smaller version of the other! We also need to know that the sides are congruent also.

Identifying congruent triangles

How can we know if two triangles (or any polygon for that matter) are congruent? We show that all corresponding parts are congruent. In other words we identify corresponding angles and corresponding sides and demonstrate they are congruent.

Example – Pg 183, Problem #24

$\Delta TRK \; and \; TUK$ …this is interesting because the triangles share a side…



First, list corresponding angles. Be careful here! Some vertices have more than one angle!:

$\angle RTK \;\&\; \angle UTK, \angle R \;\&\; \angle U, \angle RKT \;\&\; \angle UKT$

Now list corresponding sides: $\overline{TR} \;\&\; \overline{TU}$, $\overline{RK} \;\&\; \overline{UK}$, $\overline{TK} \;\&\; \overline{TK}$

Now determine if all corresponding parts are congruent. By markings and the reflexive property of congruence ($\overline{TK} \cong \overline{TK}$), we see that:

$\angle RTK \cong \angle UTK, \angle R \cong \angle U$ and $\overline{TR} \cong \overline{TU}, \overline{RK} \cong \overline{UK}, \overline{TK} \cong \overline{TK}$ …we have two angle pairs and all three sides congruent. What about the third angle pair? Let's put Theorem 4-1 to work: we know that 2 angles of the one triangle are congruent to 2 angles of the other. The theorem allows us to say the 3^{rd} angles are congruent! We're there! QED.

Thus we can say $\Delta TRK \cong TUK$

Lesson 4-1: Congruent Figures

Example – Pg 183, #26

Using the markings provided in the diagram:

Sides: There is no information given in the diagram that allows us to determine congruent sides.

Therefore, we can not conclude the triangles are congruent.

Assign homework

p. 182 #1-27 odd, 30-35, 38-40, 44, 46, 47